The QCD critical point via Padé and multi-point Padé resumations (EHPC-EXT-2022E01-055)

Christian Schmidt





HotQCD Collaboration:

Dennis Bollweg, David Clarke, Jishnu Goswami, Olaf Kaczmarek, Frithjof Karsch, Swagato Mukherjee, Peter Petreczky, CS, Sipaz Sharma

[PRD 105 (2022) 7, 074511, arXiv: 2202.09184]





Bielefeld Parma Collaboration:

David Clarke, Petros Dimopoulos, Francesco Di Renzo, Jishnu Goswami, Guido Nicotra, CS, Simran Singh, Kevin Zambello [PRD 105 (2022) 3, 034513, arXiv: 2110.15933]

Brussels, December 11, 2023

What is it all about :



Oscillatory integrals



Fundamental building blocks of matter:



1960-1964 M. Gell-Mann, G. Zweig, introducing the quark model Nobel price 1969: M. Gell-Mann

The phase diagram of QCD

- QCD thermodynamics is a consequence of spontaneous chiral symmetry breaking and liberation of new degrees of freedom: quarks
- The QCD phase diagram is encoded in the QCD partition function and can be calculated by lattice QCD at small baryon number density.



The QCD phase diagram

- QCD thermodynamics is a consequence of spontaneous chiral symmetry breaking and liberation of new degrees of freedom: quarks
- The QCD phase diagram is encoded in the QCD partition function and can be calculated by lattice QCD at small baryon number density.
- Important Landmark: A (conjectured) critical point, giving rise to a diverging correlation length and universal scaling.



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The QCD phase diagram

- The early universe passed the QCD phase transition (at small baryon number density).
- The QCD phase diagram is studied in nucleus-nucleus collisions. In particular the "Beam Energy Scan" program at RHIC is partly motivated by the search for the QCD critical point.



Relativistic Heavy Ion Collider at BNL



Lattice QCD

• The QCD partition function as path integral:

Fermion matrix: large, sparse, band structure is induced by the discretisation of the kinetic terms (derivative), the discretisation is improved by smearing and next-to-next-to nearest interactions

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Lattice QCD

• The QCD partition function as path integral:



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Simulation strategie

Calculate derivatives of the pressure



[Allton et al. PRD 66 (2002)]

perform a Padé resummation to obtain the complex singularity that limit the radius of convergence [De Frorcrand, Philipsen (2002); D'Elia, Lombardo (2003)]

obtain a rational approximation of the data (e.g. by the multi-point Padé) to obtain the closest singularity

alternatively, analyse the (asymptotic) behaviour of the Fourier coefficients

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Preliminary results I



Analysis steps:

- Perform Padé or multipoint Padé to obtain a rational approximation of the data
- Analyse poles of the rational approximation
- Identify closest pole with the Lee-Yang edge singularity

Preliminary results II



Analysis steps:

- Perform Padé or multipoint Padé to obtain a rational approximation of the data
- Analyse poles of the rational approximation
- Identify closest pole with the Lee-Yang edge singularity
- Tack the singularity as function of temperature
- Extrapolate to the critical point using universal scaling behaviour

Data is consistent with universal scaling

SIMULATeQCD

Multi-GPU code developed by our group

- Modern C++ code, uses C++17 features
- Memory management class
- Uses peer-to-peer for inter-node communication
- Uses MPI for communication between notes
- Gauge field generation by Rational Hybrid Monte Carlo
- Multi-shift and multi-RHS conjugate gradient inverter for HISQ fermion matrix
- Code available on GitHub: <u>https://github.com/LatticeQCD/SIMULATeQCD</u> [Mazur et al, [HotQCD Collaboration] arXiv:2306.01098]

The code is used on various HPC systems

- Piz Daint@CSCS: 21 PFlop/s
- Juwels-Booster@JSC: 44 PFlop/s
- Summit@ORNL: 122 PFlop/s

- Leonardo@CINECA: 238 PFlop/s
- LUMI@CSC: 379 PFlop/s

GPUs: Nvidia, AMD



Summit

Leonardo



SIMULATeQCD

- Spend most of the time in the Matrix*Vector application
- Arithmetic intensities are low: 0.73 (FP32), 0.36 (FP64)
- Increase Intensities by introducing multiple () RHS



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New physics in the muon magnetic moment?



EuroHPC

Project: "EHPC-EXT-2023E02-063"

EuroHPC used: Leonardo Booster

Speaker: Bálint TÓTH (University of Wuppertal)



Budapest-Marseille-Wuppertal (BMW) Collaboration

Sz. Borsa'nyi, Z. Fodor, J. N. Gu'nther, C. Hoelbling, S. D. Katz, L. Lellouch, T. Lippert, K. Miura, L. Parato, K. K. Szabo', F. Stokes, B. C. To'th, Cs. To'ro'k, L. Varnhorst

- Nature 593 (2021) 7857, 51-55 [arXiv:2002.12347]
- We acknowledge PRACE for awarding this project access to HAWK hosted by the High-Performance Computing Center Stuttgart.
- The computer time for this work were provided in part by the Gauss Centre for Supercomputing on the machines JUWELS, SUPERMUC and HAWK.

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The muon

- Elementary particle
- Created by cosmic rays entering the atmosphere
- ≈ 207 times heavier than electron, same charge and spin
- Acts like a tiny magnet

How strong is its magnetic moment? $g_{\mu} = ?$

- Can be measured experimentally
- Can be computed from the Standard Model (SM)
- If theory disagrees with experiment −→ New physics?



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HVP from latt

Summary & Outlook

Experiment

- Brookhaven 2004: $a_{\mu}^{BNL} = 11659209.1(6.3) \times 10^{-10}$
- Fermilab 2023: $a_{\mu}^{\text{FNAL}} = 11659205.5(2.4) \times 10^{-10}$
- Combined: $a_{\mu}^{\text{exp.}} = 11659205.9(2.2) \times 10^{-10}$

Precision: bathroom scale sensitive to weight of a single eyelash.





Muon storage ring at Fermilab

Summary & Outlook

Theory: Standard Model (SM)

Sum over all known physics:

- quantum electrodynamics (QED): photons, leptons
- 2 electroweak (EW): W, Z bosons, neutrinos, Higgs
- strong (QCD): quarks and gluons

Hadronic vacuum polarization (HVP) of photon:



dominates theory uncertainty



Two methods:

e+e- HVP

Needs $e^+ e^- \rightarrow$ hadrons cross section data

Lattice HVP

Ab initio method

this work

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Current status



e+e- HVP

Differs by more than 5σ from Fermilab.

e+e- HVP using CMD3 data

- Removes discrepancy with Fermilab.
- Not yet scrutinized by e+e- HVP community.

Lattice HVP

First full computation was done by our group. No strong tension with Fermilab, only 1.7σ .

[Plot from Run2/3 announcement on 10th Aug 2023 at Fermilab]

HVP-window

- Part of HVP: Restrict correlator to window between

 $t_1 = 0.4 \text{ fm}$ and $t_2 = 1.0 \text{ fm}$

 [RBC/UKQCD'18]

 FHM'23 [2301.08274]

 Easily computable on lattice
 - Can be computed from e+e- HVP as well
 - Latest result from each group → consensus within lattice community
 - Significant discrepancy between e+e- HVP and lattice





- Lattice gauge theory: systematically improvable, non-perturbative, 1st principles method
- Discretize space-time with lattice spacing: a



quarks on sites, gluons on links
 discretize action + operators

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 d⁴x -→ a⁴,
 ∂_µ -→ finite differences

 To get physical results, need to perform:

 Infinite volume limit (V → ∞)

Continuum limit $(a \rightarrow 0)$

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Lattice QCD		
Integrate over all class	ical field configurations $\int [\mathrm{d} U] [\mathrm{d} \overline{\psi}] [\mathrm{d} \psi] \ O \ e^{-S_{\mathfrak{g}}(\mathcal{U})-\overline{\psi}}$	Μ(U) ψ
E.g. 176 ³ × 264 lattice	$\neg \rightarrow \approx 5 \cdot 10^{10}$ dimensional integral	

Stochastic integration



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Finer/larger lattices		



- Largest source of systematic uncertainty: continuum extrapolation
- Proposed ensemble at a = 0.036 fm, $176^3 \times 264$ can further constrain fits



HVP-window:

- Agreement among different lattice groups
- Disagreement between e+e- HVP and lattice
- $\ensuremath{ \rightarrow }$ Has to be understood before New Physics can be announced or ruled out

Experiment:

- Important to pursue $e^+ e^- \rightarrow hadrons$ experiments
- MUonE experiment will provide data relevant for further cross-checks
- Lattice:
 - To keep up with future precision of Fermilab experiment factor 4x reduction of uncertainty on HVP is needed
 - Improve continuum extrapolation ← finer lattice

What do spin glasses have to say about quantum optimization?



EuroHPC

Project: "Quantum spin glasses on the GPU"

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EuroHPC used: MeluXina and Leonardo (LEAP)

Collaborators

arXiv:2310.07486



Massimo Bernaschi



Víctor Martín Mayor



Giorgio Parisi

Computational problems

- Let us consider N cities:
 - Easy problem: sort by name

CPU time scales as **N log N**

 Hard problems: organize a train trip along them

Minimize cost, path length...

• NP problem

Computational problems





Conclusions

Spin glasses: a NP problem

• Energy:

$$H = -\sum_{\langle i,j \rangle} J_{ij} s_i s_j$$



Computational problems

Quantum Optimization

Extensive GPU simulations

Conclusions

Spin glasses: a NP-complete problem

Free-energy

• Energy:

$$H = -\sum_{\langle i,j \rangle} J_{ij} s_i s_j$$

• Minimize the energy

NP-complete



Configuration space, Ω

Computational problems

Quantum Optimization

Extensive GPU simulations

Conclusions
Quantum Computing

"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical."

R. Feynman



Computational problems

Quantum Optimization

Extensive GPU simulations

Quantum spin glasses



Computational problems

Quantum Optimization



Computational problems

Quantum Optimization

Extensive GPU simulations

Extensive GPU simulations



Computational problems

Quantum Optimization

Extensive GPU simulations

Exact diagonalization

- Diagonalize Transfer Matrix T
- **T** is a $2^{L \times L} \times 2^{L \times L}$ matrix:
 - $L = 5 \rightarrow 2^{25} \times 2^{25}$
 - $L = 6 \rightarrow 2^{36} \times 2^{36}$
- Using Parity:

$$L = 6 \rightarrow 2^{35} \times 2^{35}$$
 At

- We used PetSc and Slepc libraries
- (Very) custom matrix-vector multi-GPU-CPU product for Lanczos algorithm



least 16 GPUS!

• Parallel Tempering: (k_1, k_2, \dots, k_n)



Extensive GPU simulations

• Parallel Tempering: (k_1, k_2, \dots, k_n)



Computational problems

Quantum Optimization

Extensive GPU simulations

- Parallel Tempering: (k_1, k_2, \dots, k_n)
- Checkerboard decomposition
- MUlti-SIte multispin coding
- 32-bit words





Extensive GPU simulations

- Parallel Tempering: (k_1, k_2, \dots, k_n)
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- 32-bit words





Quantum Optimization

Extensive GPU simulations

- Random bits for MUSI (briefly):
 - 1. Implement a CUDA version of philox_4x32_10 D.E. Shaws
 - (x4) 32-bit words "50/50"
 - 2. philox_4x32_10 + xoroshiro128++ S. Vigna
 - (x4) 64 bits "50/50"
 - 3. Transform 64 "50/50" bits into 8 bits with desired probs.
 - > (philox_4x32_10 + xoroshiro128++) \rightarrow 32-bits
- > 0.5 ps per spin update!

2.5 core-Mhours in Meluxina GPU+ ~3 GPU-Mhours in Leonardo

Monte Carlo simulations (main result)



Computational problems

Quantum Optimization

Extensive GPU simulations

Conclusions

We studied Quantum Phase Transition in D=2:

$$k_{c} = 0.2905(5), \qquad \frac{1}{v} = 0.70(24)(9),$$
$$\frac{\gamma^{(2)}}{v} = 0.27(8)(8), \qquad \frac{\gamma^{(3)}}{v} = 1.39(23)(11)$$

- Parity symmetry splits the configuration space.
- There are two dynamic exponents:

$$z_e = 2.46(17), \quad z \sim \infty$$

What do spin glasses have to say about quantum optimization? There are no theoretical restrictions when crossing the Quantum Phase Transition in Quantum Annealing



EuroHPC

Project: "Quantum spin glasses on the GPU"

EuroHPC used: MeluXina and Leonardo (LEAP)

What next?

Focus on hard problems (samples).

Simulate only the even sector.

> Add second-neighbor interaction.

EuroHPC

What happens at D=3?

> Go from Γ_c to $\Gamma = 0$.

Multi-scale microstructure modeling for corrosion and hydrogen embrittlement



Project: *Microstructure modeling*

EuroHPC used: LUMI-C

Speaker: Alexander Mavromaras, Materials Design

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Executive Summary

- Industry Challenges: Corrosion and hydrogen embrittlement pose significant challenges across numerous industries
- **Mitigation Strategies** necessitate a comprehensive understanding and control of the material's microstructure
- The Value of Simulations: Multi-scale simulations insights and data where experiments are costly and time-consuming
- **Collaborative Approach:** Materials Design is developing a multi-scale modeling approach together with industrial and academic partners.
- Case Study: The evolution of microstructure during the oxidation of metals, such as zirconium alloys







Overview

Company Snapshot

Materials Design - A Brief Overview of the Company and Its Technology.

Material Challenges

Exploring Corrosion and Hydride Formation in Zirconium Alloys.

• Bridging the Gap

Scaling Up from Atomistic Simulations to Microstructure Modeling.



Materials Design Company Profile

Established Legacy: Since 1998. Serving 700+ institutions worldwide.
Our Mission: Creating Engineering Value from Materials Simulations.
Offerings: MedeA® software, support, consulting, and contract research.
International Presence: Headquartered in San Diego, USA, and Paris, Europe.
Partnerships: Collaborating with technology and business partners globally.
Expertise: Computational materials science, chemistry, chemical engineering.



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Improved Decision-Making Through Data And Understanding





MedeA Environment





Corrosion

The cost of corrosion is estimated at 3.4% of the global GDP

- Water-cooled nuclear reactors: Corrosion of cladding of fuel rods is a life-limiting degradation mechanism
- Oil & Gas: Corroding pipelines drive up costs
- Automotive: Poor corrosion resistance of Magnesium alloys prevent lighter cars

Can we simulate corrosive processes over days/months/years and make predictions?









Zirconium Corrosion And Hydride Formation In Reactors

© Materials Design 2023

• Water reacts with Zr cladding of fuel rods, freeing hydrogen

Corrosion reaction:

 $Zr + 2H_2O \rightarrow ZrO_2 + 2H_2$

- Formation of Zr hydride
- Hydrogen embrittlement and mechanical weakening

How does the hydride microstructure look for a given H influx, temperature, and material?



materials design

Modeling Corrosion of Zirconium Alloys Fuel Cladding – The UW-Madison Materials Degradation under COrrosion and Radiation (MADCOR) – UW–Madison (wisc.edu)



Zr and hydride phase evolution (for 0.07s at 600K) shows effect of elastic anisotropy

3D simulation with H fluxes and elastic energy at interfaces



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From Nanoseconds To Years



© Materials Design 2023

materials design

Summary

- Safety, economy, and sustainability of primary energy generation are major challenges of this century
- Understanding corrosion and material degradation requires multi-scale modeling
- Multi-scale simulations require
 - Accurate and efficient computations of chemical, thermo-mechanical, interfacial, and transport properties
 - Machine-learned potential for scale-up and complexity
 - Realistic 3D Microstructure simulations with the above input
 - Significant parallel computing resources





More Examples

Crack formation in α -Zr – m-ZrO₂ and Mg – MgO

Cracking In Materials With Different Pilling-Bedworth Ratios



Cracks forming in a α -Zr – m-ZrO₂ multi-grain simulation at high O flux Cracks forming in a Mg - MgOmulti-grain simulation at high O flux



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Optimising Ignitor Beam Properties in Proton Fast Ignition



EuroHPC

Project: "EHPC-REG-2023R01-043"

EuroHPC used: Vega, Karolina

Speaker: Paul GIBBON (Focused Energy)

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Focused Energy was founded in July 2021





Our goal: demonstrate commercially viable inertial fusion energy

Laser-driven fusion with the Proton Fast Ignition scheme*



The Proton Fast Ignition (PFI) concept comprises several distinct steps:

- long-pulse laser driver absorption by the plasma (1)
- → fuel compression (2-3)
- short-pulse laser generation and transport of a proton beam (4-5)
- 🛶 fuel ignition and burn (6)





Acceleration and

rocket effect

Absorption and

heat transport



3 Deceleration and compression



*M. Roth et al., PRL 86, 436 (2001)

HPC access through EuroHPC is helping FE to tackle key computational design challenges



*EuroHPC project: EHPC-REG-2023R01-043



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ENERGY

Performance of EPOCH and FLASH codes on Vega & Karolina



B. Fryxel et al., Ap J. 131, 273 (2000)

ЕРОСН

T. Arber et al., PPCF 57, 113001 (2015)



- Both codes exhibiti good weak scaling, so feasible to run with 10B particles/grid points on >10⁴ cores
- CPU, MPI only (OpenMP possible). Other PIC codes available exploiting GPU, parallel I/O libraries

I. Cone-in-shell simulation of DT fuel compression with FLASH



Alfonso Mateo Aguaron, Javier Honrubia (UP Madrid & FE)

Simulation details:

- 2D cylindrical geometry for hydro & laser ray-tracing
- Grid domain 1024 μm x 2048 μm; AMR with 1 μm resolution, blocksize 16x16
- ✓ Variable timestep Δt = 1.3e-13 s; 20h runtime on 512 cores

Mitigation of FLASH technical issues:

- grid remapping to remove numerical Rayleigh-Taylor instabilities
- corrected equation of state to avoid negative pressures etc.
- smoothing across material interfaces
- calibration of shock wave propagation via cross-code benchmarking with MULTI-IFE and DUED



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II. Proton beam conversion efficiency (CE) modelling



Valeria Ospina-Bohorquez Laser parameters: DUED radintensity - $D_{hemi} = 1500 \,\mu\text{m}$ hydro $\tau_L = 3 \ ps$ contrast $\theta = 30^{\circ}$ simulations - $I_L = 3 \times 10^{19} \,\mathrm{W cm^{-2}}$ duration, shape spot size, distribution Θ wavelength? $d_{Ti} = 5 \ \mu m \ d_{CH} = 1 \ \mu m$ **Target parameters:** substrate thickness 30 Conversion efficiency Proton layer composition proton layer thickness 25 proton layer composition 20 $(LiH, CH_n, ErH_3 ...)^*$ [%] 15 +6% At today's prices, each 1% improvement in CE translates to saving of ~ \$50M in the ignitor laser system! CH CH2

ErH3

Electron

2-D simulations with diagnostic probes to characterize proton beam



 Upcoming experimental campaign on proton focusing in spring 2024 at Colorado State University (LaserNetUS Program) FOCUSED

ENERGY
III. Proton beam focusing with 'integrated' cone targets*



Javier Honrubia





Multiple effects of cone wall and DT fuel plasma:

- Strong return currents through cone walls and from DT plasma replenish foil electrons and suppress sheath field, reducing proton conversion efficiency
- Magnetic fields generated near cone tip cause strong proton beam defocusing
- Mitigation measures: reduced laser intensity, double cone walls, heavy ions
- Does the cone-tip B-field & defocusing effect still persist for mm-scale cones?

*Honrubia, Morace and Murakami, MRE **2**, 28 (2017)

Putting the pieces together for ignition-scale targets



Novel features:

- Multi-beam laser irradiation in mm-scale cone geometry: $5 \times I_L = 3.0 \times 10^{19} \text{ Wcm}^{-2}$; $\lambda = 1 \ \mu m$; $\tau_L = 3 \text{ps}$; $\sigma_{FW} = 100 \ \mu m$
- Utilize 'best of' parametric target scans: rad-hydro computed pre-plasma, laser profile, foil composition & dimensions

Numerics:

- → $30k \times 30k = 9 \times 10^8$ grid points; $\Delta x = \lambda_L/20$
- → 2 x 10⁹ particles
- → 36h on 3k cores of Vega

Future refinements:

collisions, ionization, wall isolation, 3D!



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J/cm³ $\rho/[100 \text{ g/cm}^3]$ PETRA hybrid code*: ed 0.0E+00 1.3E+17 2.7E+17 4.0E+17 4.4E+00 8.8E+00 1.3E+01 rhoT 1.0E-01 Energy deposition $\theta_{1/2} = 10^{\circ}$ $\theta_{1/2} = 10^{\circ}$ TNSA proton beam with 60 60 $T_p = 5$ MeV transported 40 40 into imploded DT Radius [um] Radius [um] 20 20 $\rho_{max} = 512 \text{ g/cm}^3$ Ω standoff distance = 1 mm -20 -20 -40 -40 E_{ig} = 18 kJ , $\theta_{1/2}$ = 0° -60 -60 $E_{ig} = 27 \text{ kJ}$, $\theta_{1/2} = 10^{\circ}$ 50 150 200 50 150 200 100 100 0 0 Depth [um] Depth [um]

IV. Proton beam divergence leads to higher ignition threshold





 T_i / keV

Towards an integrated PFI model framework







Summary

Progress on key open physics questions of Proton Fast Ignition:

- Isochoric compression of DT fuel capsule with inserted cone
- Options identified for enhancing proton beam conversion efficiency
- Proton beam focusing in full-scale cone targets: control of return currents
- Heating and ignition of compressed DT fuel: sensitivity to beam properties
- (Pre-) exascale computing resources (100s of millions of core-h) will play a vital role in de-risking inertial fusion power plant design
- Future sub-scale, high repetition-rate experimental facilities will enable quantitative calibration and refinement of models



Thanks to ...

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and

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J. J. Honrubia, V. Ospina-Bohorquez, A. Mateo-Aguaron, S. Atzeni, M. Brönner, L. Savino, X. Vaisseau, D. Callahan, W. Theobald, P. Patel, M. Roth